Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

2 - 7 Function Values

Find e^z in the form u + i v and Abs[e^z] if z equals

3. $2 \pi I (1 + I)$

```
ComplexExpand[e^(2 π I (1+I))]
```

$e^{-2\pi}$

```
N[%]
```

0.00186744

5. $2 + 3 \pi I$

```
ComplexExpand[e^(2+3 π I)]
```

$-e^2$

```
N[%]
```

-7.38906

7. $\sqrt{2} + \frac{1}{2} \pi i$

```
ComplexExpand[e^(√2 + 1/2 π i)]
```

$i e^{\sqrt{2}}$

```
N[%]
```

0. + 4.11325 i

8 - 13 Polar Form. Write in exponential form, numbered line (6), p. 631:

9. $4 + 3 I$

```
Clear["Global`*"]
```

```
z = 4 + 3 I
4 + 3 i
```

Restating the polar form described in numbered line (6),

```
Abs[z] e^(i Arg[z])
```

$5\, e^{i\, \text{ArcTan}\left[\frac{3}{4}\right]}$

```
N[Arg[z]]
0.643501
```

11. - 6.3

```
Clear["Global`*"]
```

This one takes a little “identity crisis”, as shown in numbered line (8) on p. 631,

$z == -6.3 == 6.3\,(-1) == 6.3\left(e^{\pi i}\right)$

```
True
```

13. 1 + I

```
Clear["Global`*"]
z = 1 + I
1 + i
Abs[z] e^(i Arg[z])
```

$\sqrt{2}\, e^{\frac{i\pi}{4}}$

14 - 17 Real and Imaginary Part. Find Re and Im of

15. $\text{Exp}\left[z^2\right]$

This problem is handled manually mostly.

```
Clear["Global`*"]
Expand[Exp[z^2] /. z → (x + i y)]
```

$e^{(x+i\,y)^2}$

```
int1 = Exp[Expand[(x + i y)^2]]
```

$e^{x^2+2\,i\,x\,y-y^2}$

```
int1 == Exp[x^2 - y^2] Exp[2 ⅈ x y]
```

```
True
```

Because of identity in numbered line (5) on p. 631 I can write,

```
int2 == Exp[x^2 - y^2] (Cos[2 x y] + ⅈ Sin[2 x y]);
```

And therefore, just splitting up the expression,

```
realz == Exp[x^2 - y^2] (Cos[2 x y]);
```

```
imagz == Exp[x^2 - y^2] ( Sin[2 x y]);
```

17. $\text{Exp}[z^3]$

```
Clear["Global`*"]
```

```
Expand[Exp[z^3] /. z → (x + ⅈ y)]
```

$e^{(x+i y)^3}$

```
int1 = Exp[Expand[(x + ⅈ y)^3]]
```

$e^{x^3+3 i x^2 y-3 x y^2-i y^3}$

In order to apply numbered line (5), I need to isolate terms containing i,

```
int1 == Exp[x^3 - 3 x y^2] Exp[3 ⅈ x^2 y - ⅈ y^3];
```

And then I can apply the identity,

```
int2 == Exp[x^3 - 3 x y^2] (Cos[3 x^2 y - y^3] + ⅈ Sin[3 x^2 y - y^3]);
```

so that

```
realz = Exp[x^3 - 3 x y^2] (Cos[3 x^2 y - y^3]);
```

```
imagz = Exp[x^3 - 3 x y^2] ( Sin[3 x^2 y - y^3]);
```

In this case the text does not give an answer for the imaginary part.

19 - 22 Equations. Find all solutions and graph some of them in the complex plane.

19. $e^z = 1$

The below effort looks stupid, but it’s all I could come up with.

```
Clear["Global`*"]
```

```
gs[z] = Exp[z]
```

e^z

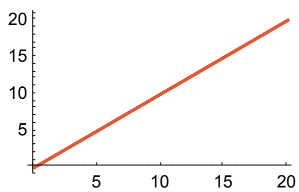
```
myt = Solve[gs[z] == 1, z]
{{z → ConditionalExpression[2 ⅈ π C[1], C[1] ∈ Integers]}}

myt2 = myt /. C[1] → d

{{z → ConditionalExpression[2 ⅈ d π, d ∈ Integers]}}

myt3 = Table[myt2, {d, 0, 10}]
{{{z → 0}}, {{z → 2 ⅈ π}}, {{z → 4 ⅈ π}},
 {{z → 6 ⅈ π}}, {{z → 8 ⅈ π}}, {{z → 10 ⅈ π}}, {{z → 12 ⅈ π}},
 {{z → 14 ⅈ π}}, {{z → 16 ⅈ π}}, {{z → 18 ⅈ π}}, {{z → 20 ⅈ π}}}

Plot[myt3, {z, 0, 20}, ImageSize → 150]
```




```
myt4 = Flatten[myt3]
{z → 0, z → 2 ⅈ π, z → 4 ⅈ π, z → 6 ⅈ π, z → 8 ⅈ π, z → 10 ⅈ π,
 z → 12 ⅈ π, z → 14 ⅈ π, z → 16 ⅈ π, z → 18 ⅈ π, z → 20 ⅈ π}

ListPlot[Table[{d, Exp[2 ⅈ d π]}, {d, 0, 10}],
 AxesLabel → {"Re", "Im"}, ImageSize → 200]
```